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Optimization of a rectangular profile annular fin based on fixed fin height†

Hyung-Suk Kang*

Division of Mechanical and Mechatronics Engineering, Kangwon National University, Chuncheon, 200-701, Korea

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Abstract

The optimum performance and fin length of a rectangular profile annular fin are presented using a variations separation method. For fixed fin height, the optimum fin length and efficiency are arbitrarily defined as those for which the heat loss is in the range between 90% and 99% of the maximum heat loss. The maximum heat loss, the maximum effectiveness, the minimum fin resistance, the optimum fin length and the optimum efficiency are presented as a function of the inside fluid convection characteristic number, fin base thickness, fin height and ambient convection characteristic number. One of the results shows that the optimum fin length decreases almost linearly with the increase of the fin base thickness.

***Keywords*:** Annular fin; Convection characteristic number; Fin effectiveness; Fin length; Heat loss

1. Introduction

Extended surfaces or fins have been considered as one of the most effective means of enhancing the rate of heat transfer between a primary surface and its surrounding fluid in many engineering and industrial applications. Annular fins are one of the important components of the finned tube heat exchanger, automobile and aerospace and so on. A number of publications on the performance of an annular fin are available in the literature. For example, assuming the boundary conditions of constant temperature at the fin base and insulation at the fin tip, the performance of an annular fin of rectangular profile is reported [1]. The performance of eccentric annular disk fins with a variable base temperature is analyzed using a bipolar coordinate transformation [2]. Also, a finite difference procedure is employed to analyze the effectiveness of radiating-convecting fins [3], while Lalot et al. [4] develop an expression for the temperature distribution and efficiency of annular fins made of two materials.

Recently, the optimization procedure for the fin shapes has been studied more vigorously. One of the representative ways is to fix a suitable profile, and then determine the dimensions of the fin which yields the maximum heat dissipation for a given fin volume or mass. For example, the optimum design of single longitudinal fins is presented by means of an accurate mathematical method [5]. The optimal pin fin array of variable cross section for a given fin material per unit base area is investigated [6]. A reversed trapezoidal fin with variable fin base thickness is analyzed and optimized using a two-dimensional analytic method [7]. Also, there are many of these kinds of optimizations for the annular fins. For example, Yu and Chen [8] discuss the optimization of rectangular profile circular fins with variable thermal conductivity and convective heat transfer coefficients. The optimum procedure for a thermally asymmetric convective and radiating annular fin is presented [9]. Kundu and Das [10] develop a comprehensive scheme for optimization of elliptic fins.

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*Corresponding author. Tel.: +82 33 250 6316, Fax.: +82 33 242 6013
E-mail address: hkang@kangwon.ac.kr

It seems to be more realistic to optimize the fins based on the fixed fin height instead of fixed fin volume. For this kind of optimization, Look [11] presents the effectiveness as a function of Biot number for the annular fin on a pipe. A geometrically asymmetric trapezoidal fin under thermally asymmetric condition is optimized [12]. In this paper, the optimization, based on fixed fin height, of the rectangular profile annular fin with fluid in the pipe is presented. For a fin base boundary condition, convection from inside fluid to the pipe inside wall, conduction from the pipe inside wall to the fin base and conduction through the fin base are simultaneously considered. Also, heat convection from the fin tip is not ignored. Under these fin base and tip boundary conditions, the maximum heat loss, the maximum effectiveness, the minimum fin resistance, the optimum fin length and the optimum efficiency are presented as a function of the inside fluid convection characteristic number, fin base thickness, fin height and ambient convection characteristic number.

2. 2-D Analytical methods

2.1 Heat loss

The two-dimensional governing differential equation for an annular rectangular profile fin as shown in Fig. 1 can be written

$$\frac{\partial^2\theta}{\partial R^2}+\frac{1}{R}\frac{\partial\theta}{\partial R}+\frac{\partial^2\theta}{\partial Z^2} = 0 \tag{1}$$

Four boundary conditions simultaneously required to solve Eq. (1) are listed as Eqs. (2)-(5).

$$\left.\frac{\partial\theta}{\partial Z}\right|_{Z=0} = 0 \tag{2}$$

$$\left.\frac{\partial\theta}{\partial R}\right|_{R=R_e} + M\left.\theta\right|_{R=R_e} = 0 \tag{3}$$

$$-\left.\frac{\partial\theta}{\partial R}\right|_{R=R_b} = \frac{1-\left.\theta\right|_{R=R_b}}{\dfrac{R_b}{M_f}+R_b\ ln\left(R_b\right)} \tag{4}$$

$$\left.\frac{\partial\theta}{\partial Z}\right|_{Z=L} + M\left.\theta\right|_{Z=L} = 0 \tag{5}$$

When Eq. (1) with three boundary conditions [Eqs. (2)-(4)] is solved, the dimensionless temperature distribution of the fin can be expressed as

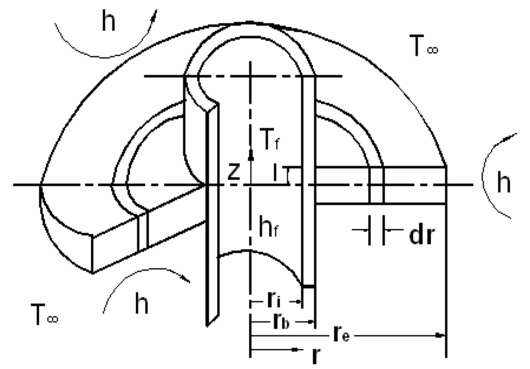

Fig. 1. Geometry of a rectangular profile annular fin.

$$\theta\left(R,Z\right)=\sum_{n=1}^{\infty}\frac{A_n}{B_n+C_n\cdot D_n}f\left(R\right)f\left(Z\right) \tag{6}$$

where,

$$A_n = \frac{4sin\left(\lambda_n L\right)}{2\lambda_n L+sin\left(2\lambda_n L\right)} \tag{7}$$

$$B_n = I_0\left(\lambda_n R_b\right)+f_n\ K_0\left(\lambda_n R_b\right) \tag{8}$$

$$C_n = \lambda_n\left\{R_b/M_f+R_b\ ln\left(R_b\right)\right\} \tag{9}$$

$$D_n = \left\{f_n\ K_1\left(\lambda_n R_b\right)-I_1\left(\lambda_n R_b\right)\right\} \tag{10}$$

$$f\left(R\right)=I_0\left(\lambda_n R\right)+f_n\ K_0\left(\lambda_n R\right) \tag{11}$$

$$f\left(Z\right)=cos\left(\lambda_n Z\right) \tag{12}$$

$$f_n=\frac{\lambda_n I_1\left(\lambda_n R_e\right)+MI_0\left(\lambda_n R_e\right)}{\lambda_n K_1\left(\lambda_n R_e\right)-MK_0\left(\lambda_n R_e\right)} \tag{13}$$

The eigenvalues λ_n can be calculated using Eq. (14), which is arranged from Eq. (5).

$$\lambda_n\cdot\tan\left(\lambda_n\right)=M \tag{14}$$

The heat loss from the rectangular profile annular fin is calculated from Eq. (15).

$$\begin{aligned} q &= \int_{-l}^{l}-k\left.\frac{\partial T}{\partial r}\right|_{r=r_b}2\pi r_b\ dz \\ &= -k\phi_f\left(2\pi r_i\right)\int_{-L}^{L}\left.\frac{\partial\theta}{\partial R}\right|_{R=R_b}R_b dZ \end{aligned} \tag{15}$$

Dimensionless heat loss is written as

$$Q=-4\pi\sum_{n=1}^{\infty}\frac{A_n\cdot E_n}{B_n+C_n\cdot D_n}sin\left(\lambda_n L\right) \tag{16}$$

where,

$$E_n = I_1(\lambda_n R_b) - f_n K_1(\lambda_n R_b) \tag{17}$$

2.2 Fin effectiveness

Fin effectiveness is defined as the ratio of heat loss from the fin to that from the bare pipe which has the same area as the fin base. If the heat transfer to the z direction is ignored between pipe inside wall and pipe outside wall (i.e., from r_i to r_b) of the bare pipe, the dimensionless energy balance equation for the bare pipe can be written as

$$R\frac{d^2\theta}{dR^2} + \frac{d\theta}{dR} = 0 \tag{18}$$

Inside and outside wall boundary conditions for the bare pipe case are given in Eqs. (19)-(20).

$$M_f\left(1 - \theta\big|_{R=1}\right) = -\frac{d\theta}{dR}\bigg|_{R=1} \tag{19}$$

$$-\frac{d\theta}{dR}\bigg|_{R=R_b} = \mathrm{M}\cdot\theta\big|_{\mathrm{R}=R_b} \tag{20}$$

When Eq. (18) is solved using the boundary conditions of Eqs. (19)-(20), the dimensionless temperature distribution in the bare pipe is expressed by

$$\theta(R) = \frac{R_b \cdot M_f \cdot M \cdot ln(R_b / R) + M_f}{R_b \cdot M \cdot \{M_f \cdot ln(R_b) + 1\} + M_f} \tag{21}$$

The heat loss from the bare pipe is calculated as indicated by

$$q_p = -k\phi_f\,(2\pi r_i)(2R_b L)\frac{d\theta}{dR}\bigg|_{R=R_b} \tag{22}$$

Then, the dimensionless form of the heat loss from the bare pipe can be expressed by

$$Q_p = \frac{4\pi L \cdot R_b \cdot M_f \cdot M}{R_b \cdot M \cdot \{M_f \cdot ln(R_b) + 1\} + M_f} \tag{23}$$

The fin effectiveness, ε, can be expressed by Eq. (24) as previously defined.

$$\varepsilon = \frac{Q}{Q_p} \tag{24}$$

The annular fin effectiveness can be used to decide whether we will use the fin or not.

2.3 Fin efficiency

Fin efficiency is defined as the ratio of actual heat loss from the fin to the ideal heat loss from the fin. An ideal heat loss from the fin is expressed by Eq. (25) and dimensionless form is given by Eq. (26).

$$q_{id} = h\,\left\{2\pi\left(r_e^2 - r_b^2\right) + 2\pi r_e \cdot 2l\right\}\,\left(T\big|_{r=r_b} - T_\infty\right) \tag{25}$$

$$Q_{id} = 2\pi M\left(R_e^2 - R_b^2 + 2R_e L\right)\cdot\theta\big|_{X=R_b} \tag{26}$$

Then fin efficiency is written by

$$\eta = \frac{Q}{Q_{id}} \tag{27}$$

2.4 Fin resistance

Fin resistance is defined as the ratio of temperature difference between the fin base and ambient to the heat loss from the fin and is presented by

$$fr = \frac{T\big|_{x=r_b} - T_\infty}{q} \tag{28}$$

Dimensionless form of the fin resistance is given by

$$FR = fr \cdot r_i \cdot k = \frac{\theta\big|_{X=R_b}}{Q} \tag{29}$$

3. Results and discussions

Fin performance as a function of the fin length for different values of ambient convection characteristic number is presented in Fig. 2. It shows that the heat loss and effectiveness increase whereas the fin resistance and efficiency decrease as the fin length increases. Fin resistance decreases very markedly at first (i. e., 1.11< R_e < 1.5 for M =0.05 and 1.11 < R_e < 2 for M =0.02), while the variation of that is almost negligible with increase of the fin length over about R_e =4 for both of M. It can be noted that the efficiency difference between M =0.02 and M =0.05 increases at first and then decreases with the increase of fin length.

Fig. 3 presents the maximum values of heat loss, fin effectiveness and the minimum fin resistance as a

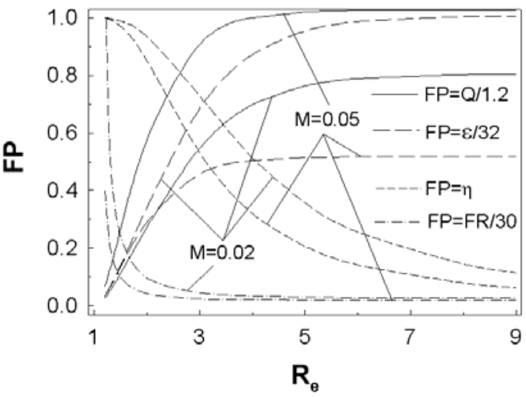

Fig. 2. Fin performance versus the fin length in the case of M_f =20, R_b =1.2 and L =0.1.

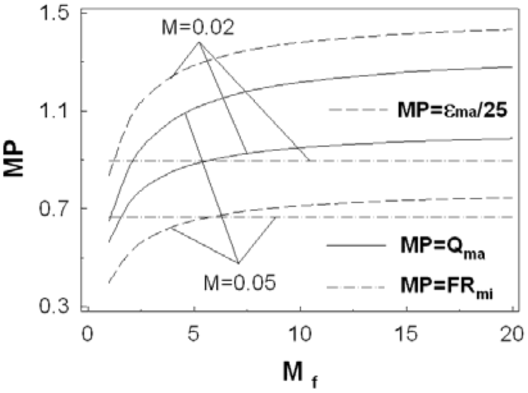

Fig. 3. Maximum (minimum) performance vs. the inside fluid convection characteristic number for R_b =1.1 and L =0.1.

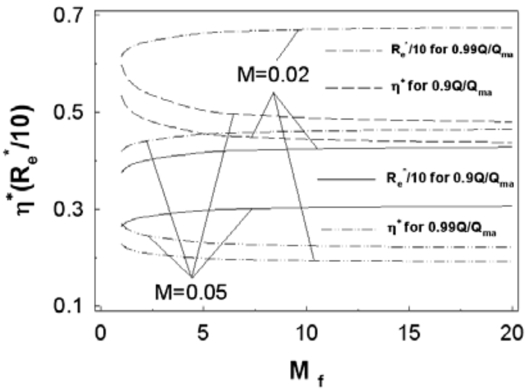

Fig. 4. Optimum values versus the inside fluid convection characteristic number for R_b =1.1 and L =0.1.

function of the inside fluid convection characteristic number when the fin height is fixed. The maximum heat loss and effectiveness increase, but the increasing rate becomes smaller with the increase of inside fluid

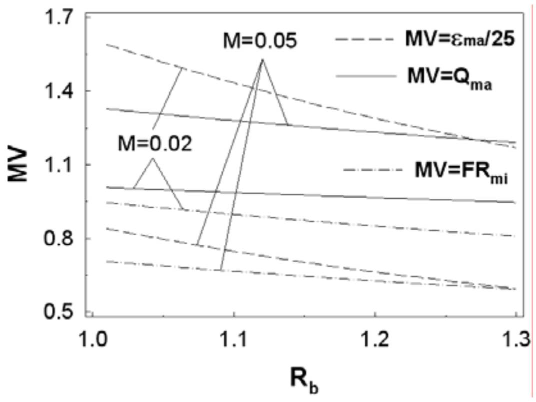

Fig. 5. Maximum (minimum) performance vs. the fin base thickness for M_f =20, L =0.1.

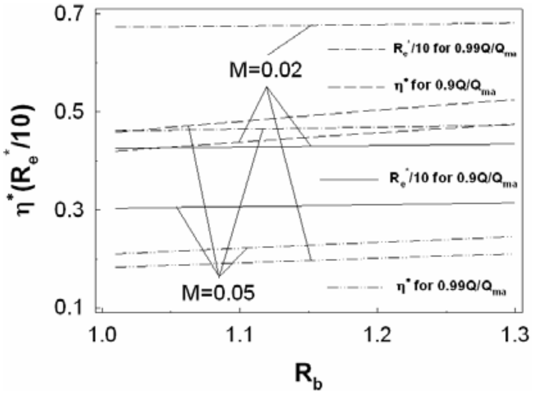

Fig. 6. Optimum values versus the fin base thickness for M_f =20, L =0.1.

convection characteristic number. It shows that the minimum fin resistance is independent of the variation of inside fluid convection characteristic number. Physically, it means that the increasing rate of dimensionless fin base temperature is the same as that of the maximum heat loss with the increase of inside fluid convection characteristic number.

Fig. 4 shows the variation of the optimum fin length and efficiency under the same conditions as given in Fig. 3. The optimum fin length is defined as the fin length which is between the fin length for $0.9\,Q/Q_{ma}$ and that for $0.99\,Q/Q_{ma}$. The optimum efficiency is defined as the efficiency at the optimum fin length. The optimum fin length increases whereas the optimum efficiency decreases with the increase of inside fluid convection characteristic number. It can be noted that the variation of optimum values with the increase of M_f is very small over bout M_f =10.

The maximum values of heat loss, fin effectiveness

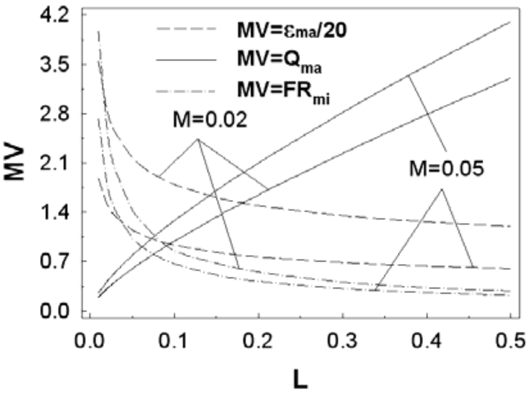

Fig. 7. Maximum (minimum) performance versus the fin height for M_f =20, R_b =1.1.

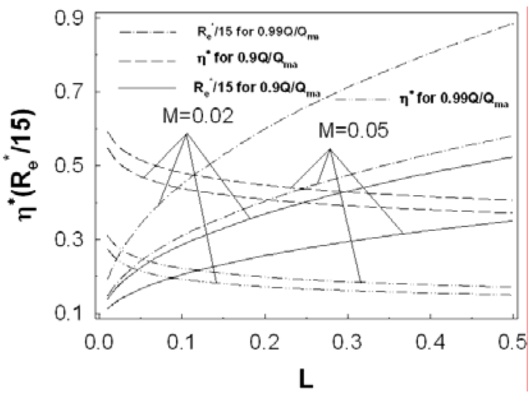

Fig. 8. Optimum values versus the fin height for M_f =20, R_b =1.1.

and the minimum fin resistance as a function of the fin base thickness are presented in Fig. 5. Both the maximum heat loss and effectiveness decrease with the increase of fin base thickness but the decreasing rate of the maximum effectiveness is more remarkable than that of the maximum heat loss. The minimum fin resistance also decreases even though the maximum heat loss decreases as the fin base thickness increases. It is because the decreasing rate of dimensionless fin base temperature is larger than that of the maximum heat loss with the increase of fin base thickness.

Fig. 6 shows the variation of the optimum fin length and efficiency with the variation of fin base thickness. The optimum fin tip radius increases very slightly as the fin base thickness increases. It should be noted that the actual optimum fin length decreases almost linearly with the increase of fin base thickness because the optimum fin tip radius minus the fin base

Table 1. Optimum fin tip radius and corresponding increasing rate of heat loss for M_f =10, L =0.15 and R_b =1.1.

P	M	R_e for $P\,(Q\,/\,Q_{ma})$	$\{Q\,(R_e+0.1)\text{-}Q\,(R_e)\}$ $/\,Q\,(R_e)(\%)$
0.9	0.01	6.5516	0.5541
	0.05	3.4579	1.2400
	0.1	2.7062	1.6919
0.99	0.01	10.7628	0.0532
	0.05	5.3834	0.1136
	0.1	4.0816	0.1559

thickness is the optimum fin length. It can be known that the corresponding optimum efficiency increases linearly with the increase of fin base thickness and the increasing rate of optimum efficiency for 0.9 Q/Q_{ma} is larger than that for 0.99 Q/Q_{ma} .

Fig. 7 depicts the maximum values of heat loss, fin effectiveness and the minimum fin resistance as a function of the fin height. The maximum heat loss increases continuously whereas the maximum effectiveness decreases very rapidly and then decreases slowly with the increase of fin height. This phenomenon means physically that the increasing rate of heat loss from the bare outer wall is larger than the increasing rate of the maximum heat loss from the fin with the increase of fin height. It shows that the minimum fin resistance decreases very rapidly at first and then levels off, and the effect of ambient convection characteristic number on the minimum fin resistance becomes smaller as the fin height increases.

The variation of the optimum fin length and efficiency with the variation of fin height is presented in Fig. 8. The optimum fin length increases somewhat rapidly first and then increases continuously, whereas the optimum efficiency decreases a little remarkably first and then decreases slowly as the fin height increases. It can be noted that the effect of ambient convection characteristic number on the optimum efficiency seems to be not much for the given range of fin height, whereas that on the optimum fin length becomes remarkable as the fin height increases.

Table 1 lists the fin tip radius with variation of P and M and corresponding increasing rate of heat loss when the fin tip radius is increased by 0.1. It can be known that the increasing rate of heat loss increases as M increases and/or P decreases. The values of increasing rate of heat loss are between 0.0532% and 1.6919% for given range of P and M so the fin tip radius can be alternatively defined as the optimum fin tip radius if the increasing rate of heat loss is between 0.05% and 1.70%. In other words,

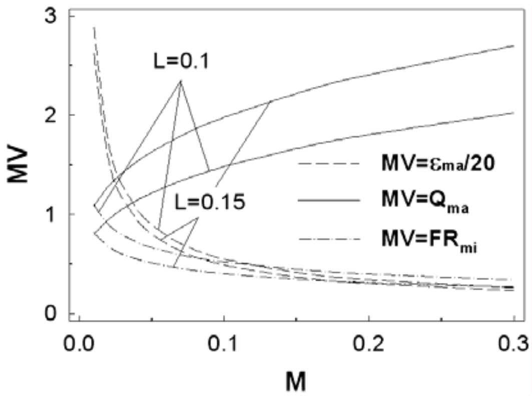

Fig. 9. Maximum (minimum) performance vs. the ambient convection characteristic number for M_f =10, and R_b =1.1.

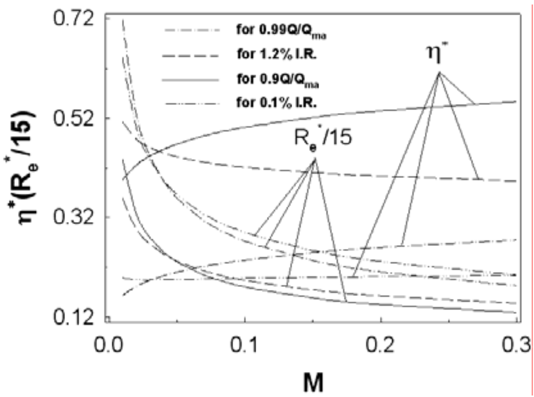

Fig. 10. Optimum values vs. the ambient convection characteristic number for M_f =10, R_b =1.1 and L =0.15.

R_e can be alternatively defined as R_e^* if the value of $\{Q(R_e+0.1)-Q(R_e)\}/Q(R_e)$ is between 0.05% and 1.7% for given range of P and M in Table 1.

The variation of the maximum heat loss, maximum effectiveness and the minimum fin resistance with the variation of ambient convection characteristic number is presented in Fig. 9. It shows that both the maximum effectiveness and the minimum fin resistance decrease in parabolic, whereas the maximum heat loss increases in parabolic with the increase of ambient convection characteristic number. It can be noted that the maximum effectiveness increases very remarkably as the ambient convection characteristic number decreases from 0.05 to 0.01. This phenomenon explains why the fin is more useful under free convection condition.

Fig. 10 shows the variation of the optimum fin length and efficiency as a function of the ambient convection characteristic number. In this figure, the optimum values as defined alternatively in Table 1 are presented and 1.2% I. R. means that the value of $\{Q(R_e+0.1)-Q(R_e)\}/Q(R_e)$ is 1.2%. The optimum fin length decreases in parabolic for all four optimum criteria as the ambient convection characteristic number increases. But the corresponding optimum efficiency increases for the former optimum criterion, whereas that decreases or almost remains as a constant for the alternatively defined optimum criterion with the increase of ambient convection characteristic number.

4. Conclusions

From the optimization of a rectangular profile annular fin based on the fixed fin height by using a variables separation method, the following conclusions can be drawn:

(1) The maximum heat loss, maximum effectiveness and the minimum fin resistance are presented as a function of the inside fluid convection characteristic number, fin base thickness, fin height and ambient convection characteristic number. One of characteristic phenomena is that the minimum fin resistance is independent of the inside fluid convection characteristic number.

(2) The optimum fin length increases as the inside fluid convection characteristic number and fin height increase or fin base thickness and ambient convection characteristic number decrease.

(3) The optimum dimensionless fin length is between about 1.70 and 10.76 and the corresponding optimum fin efficiency is between 18.98% and 59.95% for the range of all given variables in this study. The data for the optimum fin length can be used for practical fin design problems.

Nomenclature

fr	:	Fin resistance (K/W)
FR	:	Dimensionless fin resistance, $k \cdot fr \cdot r_i$
h	:	Heat transfer coefficient over the fin (W/m^2℃)
h_f	:	Inside fluid heat transfer coefficient (W/m^2℃)
I_0	:	Modified Bessel function of the first kind of order 0
I_1	:	Modified Bessel function of the first kind of order 1
k	:	Thermal conductivity of fin material (W/m℃)
K_0	:	Modified Bessel function of the second kind

of order 0

K_1 : Modified Bessel function of the second kind of order 1

l : One half fin height (m)

L : Dimensionless form of l, l/r_i

M : Ambient convection characteristic number, $(h\,r_i)/k$

M_f : Inside fluid convection characteristic number, $(h_f\,r_i)/k$

MV : Maximum or minimum value

q : Heat loss from the fin (W)

Q : Dimensionless heat loss from the fin, $q/(k\phi_f r_i)$

Q^* : Dimensionless optimum heat loss from the fin

r : Radius coordinate (m)

R : Dimensionless radius coordinate, r/r_i

r_b : Fin base radius or outer radius for a bare pipe (m)

R_b : Dimensionless form of r_b, r_b/r_i

r_e : Fin tip radius (m)

R_e : Dimensionless fin tip radius, r_e/r_i

r_i : Inside radius of the pipe (m)

T : Fin temperature (℃)

T_f : Inside fluid temperature (℃)

T_∞ : Ambient temperature (℃)

z : Fin height coordinate (m)

Z : Dimensionless fin height coordinate, z/r_i

Greek letters

ε : Fin effectiveness

ϕ_f : Adjusted inside fluid temperature (℃), $(T_f - T_\infty)$

η : Fin efficiency

λ_n : Eigenvalues (n = 1, 2, 3, ···)

θ : Dimensionless temperature, $(T - T_\infty)/(T_f - T_\infty)$

Subscripts

b : Fin base or outer radius for a bare pipe

e : Fin tip

f : Fluid inside the pipe

i : Inside wall of the pipe

id : Ideal

ma : Maximum

mi : Minimum

p : Bare pipe

∞ : Ambient

Superscript

* : Optimum

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Hyung-Suk Kang received the B.S. and M.S. in Mechanical Engineering from the Hanyang University in Seoul, Korea, in 1984 and 1986. He holds a Ph.D. from the University of Missouri-Rolla in 1992 and is presently a professor in the Division of Mechanical and Mechatronics Engineering of the Kangwon National University in Chuncheon, Korea. He is the author/co-author of more than 90 papers and proceedings